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On the shape of a deformed eddy with varying Coriolis force

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We present a simple model of a steady barotropic eddy superimposed on a current, with varying Coriolis force, to study the structure of a nonlinear deformed eddy. We also develop the linearized time-dependent formulation of the problem. The resulting solutions are stable perturbations, with scales smaller than the eddy.

1. Introduction

The lively interest aroused by the recent MODE experiments has emphasized some questions concerning mid-oceanic eddies. The origin, the energy balance and the stability of baroclinic eddies, in a region large enough for Coriolis force not to be constant, have been studied by Robinson (1975), McWilliams (1976), Huppert & Bryan (1976). Some possible mechanisms have been proposed and studied with a view to finding a reasonable model for the growth of these eddies.

We shall present a simple model, that of a barotropic eddy superimposed on a current, with varying Coriolis force, §2, to study the structure of a nonlinear eddy. We have used particular care in handling the nonlinear terms in order to give a detailed description of the various effects. To our surprise the equation for the steady case, a complicated nonlinear equation, decouples into two linear equations, the former concerning the eddy, the second relating to the current.

The problem of the effects of nonlinearity has previously interested some researchers. The interactions of various meteorological eddies have also been investigated by Friedlander (1975). In this context the linearized equations found by Friedlander have some features in common with this work. Ingersoll (1973) has numerically studied a similar case for Jupiter's atmosphere; Stern (1976) has used variational approaches to study the dimensions of oceanic vortices. Larichev & Reznic (1976) have also studied the relation between nonlinear eddies and mean flows. A recent study of a quasigeostrophic eddy (Flierl 1977) bears some relation with this note.

Assuming that the strength of the current is smaller than the eddy strength, one can also develop the time-dependent formulation of the problem, in terms of the current strength, §3–4. The resulting solutions are stable perturbations with scales smaller than the eddy. Although further development looks interesting, the difficulty of the problem increases considerably.

2. The steady case

The steady nonlinear motion of an inviscid fluid is studied. The Coriolis force $f(y)$ depends on the y co-ordinate, directed northward. The x co-ordinate is directed eastward. The centre $(0, 0)$ is a point in a mid-oceanic region.

The aim is to see if an eddy can exist with a current. The Euler equation is used for a stream function $\psi(x,y,t)$:

$$\partial_t \Delta\psi + J(\psi, \Delta\psi + f(y)) = 0. \tag{1}$$

The quantity $J(A,B)$ is the Jacobian determinant

$$J(A,B) = \partial_x A \partial_y B - \partial_x B \partial_y A = \frac{1}{r}(\partial_r A \partial_\theta B - \partial_r B \partial_\theta A).$$

Here and in the following we will alternatively use the polar co-ordinates (r,θ) and the Cartesian co-ordinates (x,y), in view of the dual nature of the problem, i.e. curvilinear and rectilinear, eddy and current.

We will proceed by steps. We start with f = constant; then every $\phi(r)$ is a solution to the steady equation,

$$J(\phi(r), \Delta\phi(r) + f) = 0$$

if $\phi(r)$ is 'fairly regular': $\phi \in G_3$.

This is no longer true if $f = f(y)$:

$$J(\phi(r), \Delta\phi(r) + f(y)) = f'(y)\,\partial_x \phi(r) \neq 0,$$

i.e. ϕ is no longer a solution. We can now assume, on heuristic grounds, that $f = f(y)$ and

$$\psi = \phi(r) + \alpha(y).$$

This gives a current over an eddy, or a deformed eddy. One could also assume currents in different directions with no further difficulty. The Euler equation is thus:

$$\begin{aligned} J(\psi, \Delta\psi + f(y)) &= \partial_x\phi(r)\,[\alpha'''(y) + f'(y)] - \alpha'(y)\,\partial_x \Delta\phi(r) \\ &= \cos\theta\,[\partial_r\phi(\alpha''' + f') - \alpha'\partial_r\Delta\phi] = 0. \end{aligned} \tag{2}$$

This equation has a solution if and only if

$$\left.\begin{aligned} \Delta[\partial_r\phi(r)] &= N[\partial_r\phi(r)], \\ \alpha'''(y) + f'(y) &= N\alpha'(y), \end{aligned}\right\} \tag{3}$$

where N is a constant.

If we assume regularity at infinity, we have $N = -K^2$. The eddy solutions are Hankel functions (Courant & Hilbert 1953); assuming regularity at the origin, the solution is a Bessel function (Flierl, 1977)

$$\phi = \tilde{A} J_0(Kr) + \tilde{B}, \quad 0 \leqslant r < \infty.$$

Its behaviour near the origin reminds us intuitively of a bell of radius $R_0 \sim 4/K$, where $\partial_r \phi \neq 0$ for $r \leqslant R_0$. At infinity it has an oscillatory behaviour.

It should be noted that our basic assumption $\psi = \phi(r) + \alpha(y)$ implies that the energies of our system cannot be finite in the x, y plane.

We can now discuss the current in the β plane or β–β plane approximation:

$$f = f_0 + \beta y + \gamma y^2.$$

The solution is

$$\alpha(y) = \tilde{\mathscr{A}} \sin Ky + \tilde{\mathscr{B}} \cos Ky - \frac{f(y)}{K^2} + \mathscr{D}.$$

The general solution is then (figure 1)

$$\psi = AJ_0(Kr) + B\sin Ky + C\cos Ky - \frac{f(y)}{K^2} + D. \tag{4}$$

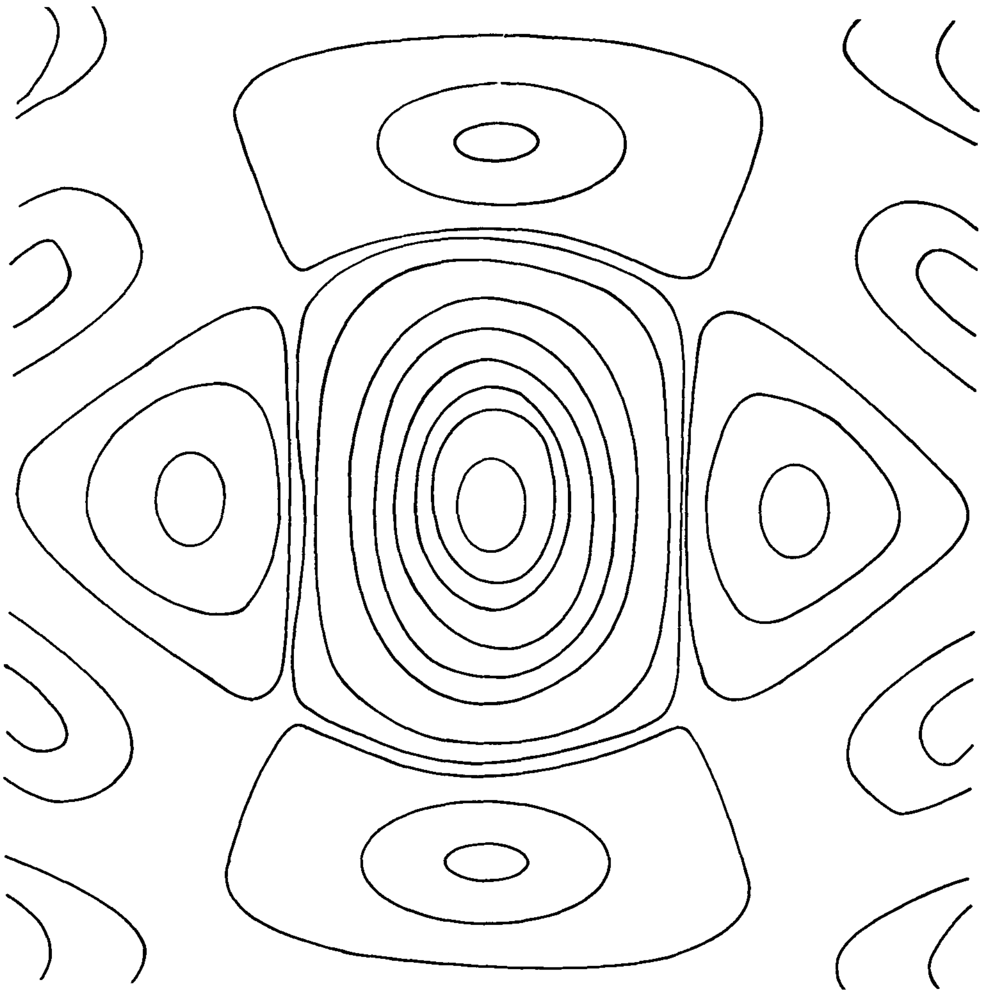

FIGURE 1. The case of a vortex with
$\psi = 0{\cdot}30 J_0(0{\cdot}9\, r) - 0{\cdot}05 \cos(0{\cdot}9\, y), f = \text{const.}$

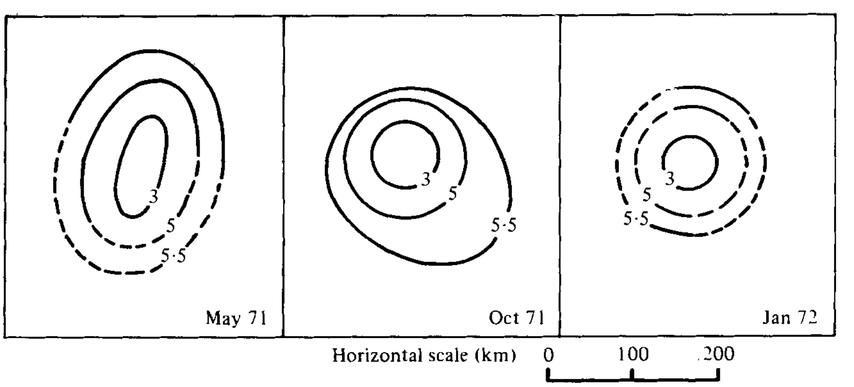

FIGURE 2. Experimental 'elliptic' vortices found by Cheney & Richardson (1976).

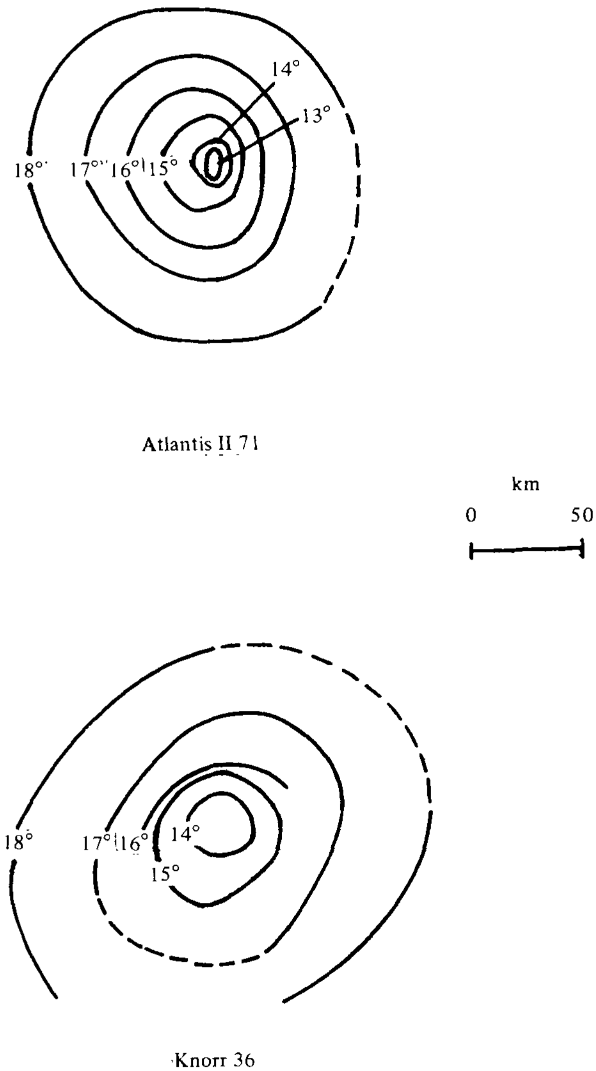

FIGURE 3. Experimental 'elliptic' vortices found by Wiebe *et al.* (1976).

The Coriolis force is compensated by a contribution

$$-\frac{f}{K^2} = -\frac{\beta}{K^2}y - \frac{\gamma}{K^2}y^2$$

to ψ.

It is perhaps interesting to observe that for $B = C = 0$, (4) is the solution of the Rossby wave equation found by Longuet-Higgins (1964), combined with an eastward current which balances the westward phase propagation.

Let us now discuss this solution on physical grounds. An interesting case of oceanic eddy has been studied by Cheney & Richardson (1976). It has been examined for a long period and its time decay has been computed (figure 2). The shape of this eddy is similar to the central part of our model (figure 1). The velocity profiles in the central part are also in reasonable agreement with our computed velocity field. In the work

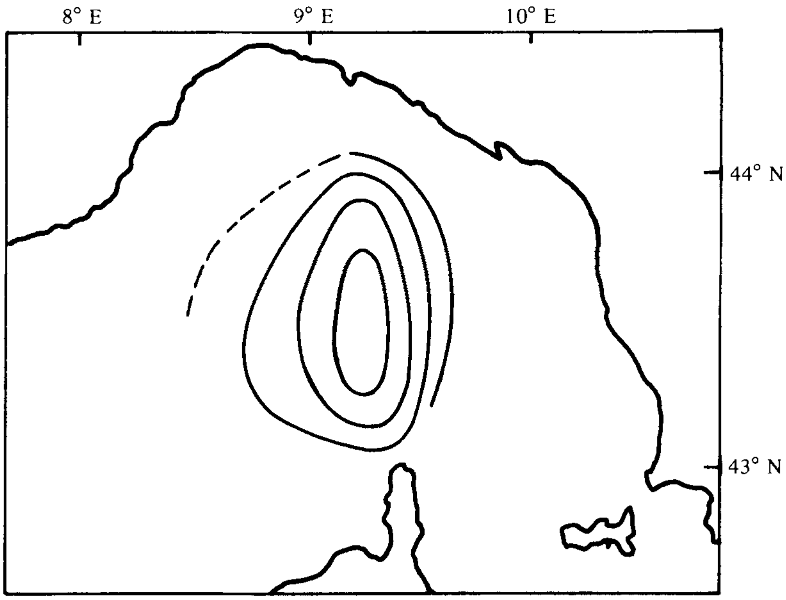

FIGURE 4. Experimental 'elliptic' vortices found by De Maio & Trotti (1967).

of Cheney & Richardson no realistic information about the external part of the eddy is available. Although there is no definitive evidence for an identification of this oceanic eddy with our $J_0(Kr)$ model, we feel that this hypothesis is reasonable. Other experimental profiles similar to that of figure 2 have been found by Wiebe (1976) (figure 3) and De Maio & Trotti (1967) (figure 4). A recent review is due to Lai & Richardson (1977).

One might however wonder why only the central part could be detected experimentally. This could happen because at some distance the effect of $J_0(Kr)$ would be a secondary quantity in comparison with the current and any other external force. From another point of view, let us remark that for $\alpha = 0$, the Rayleigh stability condition

$$\Phi \equiv \frac{d}{dr}(rV)^2 > 0$$

is not satisfied for every r $(0 \leqslant r < \infty)$. One could thus expect that this solution has a physical meaning in a central region $\mathscr{C}$ only.

3. Time-dependent variations

Let us now extend the preceding analysis by allowing a rather small time-dependent variation

$$\mathscr{D}(x,y,t) = \exp(\omega t)\, d(x,y) + \exp(\omega^* t)\, d^*(x,y).$$

Then
$$\left.\begin{aligned} \psi = \psi(x,y,t) &= \phi(r) + \alpha(y) + \mathscr{D}(x,y,t) \\ &= \phi(r) + \alpha(y) + \exp(\omega t)\, d(x,y) + \exp(\omega^* t)\, d^*(x,y). \end{aligned}\right\} \quad (5)$$

The quantity $\mathscr{D}$ is assumed to be rather small and it will be treated as a perturbation (Chandrasekar 1961; Davis 1976; Lalas 1975). One important aspect is the nature of ω: if it is a purely imaginary number, the effect of $\mathscr{D}(x,y,t)$ remains stationary.

If on the contrary it is a complex or real number, then one can assume that $\mathscr{D}$ can increase or decrease with time.

Let us remember that the experimental data of Cheney & Richardson (1976) give an average decay time of ~ 2 years for oceanic eddies of $50 \div 100$ km in radius: a particularly stable system.

We first note that the eddy $\phi(r)$ by itself does not satisfy the Rayleigh stability condition

$$\Phi \equiv \frac{d}{dr}(rV)^2 = 2K\,[J_1^2(Kr) - J_1(Kr)\,J_2(Kr)] > 0$$

for every r $(0 \leqslant r < \infty)$ but only for r inside some central region $\mathscr{C}$ whose radius is typically of order R_o. As α increases one would intuitively expect a similar result for a 'deformed' central region $\mathscr{C}$ obtained by continuous deformation of the circle $r \sim Ro$.

We thus study perturbations localized on $\mathscr{C}$ i.e. perturbations $\mathscr{D}(x, y, t)$ such that $\mathscr{D} = 0$ on the boundary $\partial\mathscr{C}$. These are not the most general kind of perturbations but in this case some interesting results are available.

The Euler equation for the perturbations is

$$\begin{aligned}\omega \exp(\omega t)\,\Delta d + \omega^* \exp(\omega^* t)\,\Delta d^* + J\{(\alpha+\phi), \Delta(\alpha+\phi)+f\} + \exp(\omega t)\,\{J(\phi, \Delta d) \\ + J(\alpha, \Delta d) + J(d, \Delta\phi) + J(d, \Delta\alpha) + J(d, f)\} + \exp(\omega^* t)\,\{J(\phi, \Delta d^*) \\ + J(\alpha, \Delta d^*) + J(d^*, \Delta\phi) + J(d^*, \Delta\alpha) + J(d^*, f)\} = O(d^2) \simeq 0\end{aligned}$$

disregarding the quadratic d terms. Since $\phi(r)$ and $\alpha(y)$ have been fixed by the steady case, we now have a linearized equation for ω and d.

The part which is proportional to $\exp(\omega t)$ is

$$\begin{aligned}\omega\Delta d + \frac{1}{r}\,\partial_r\phi\partial_\theta\Delta d - \frac{1}{r}\,\partial_r\Delta\phi\partial_\theta d - \alpha'\partial_x\Delta d + \alpha'''\partial_x d + f'\partial_x d \\ = \omega\Delta d + \frac{1}{r}\,\partial_r\phi\,\partial_\theta\Delta d - \frac{1}{r}\,\partial_r\Delta\phi\partial_\theta d - \alpha'(K^2+\Delta)\,d = 0. \qquad (6)\end{aligned}$$

Computational difficulties arise and we need some kind of approximation. If the velocities of the circular eddy are much larger than the current, we can introduce a small parameter $\epsilon \sim \alpha/\phi$ and study the equation of motion by a series expansion in ϵ. Owing to the smallness of the β-effect and of α throughout the region we want to study, we can obtain meaningful results by studying the equation (6) up to first order in ϵ or β.

We then can write:

$$\begin{aligned}&\alpha \Rightarrow \epsilon\alpha,\\ &\omega \Rightarrow \omega_0 + \epsilon\omega_1 + \ldots = \sum_k \epsilon^k \omega_k,\\ &f \Rightarrow f_0 + \beta y,\\ &d \Rightarrow d_0 + \epsilon d_1 + \ldots = \sum_k \epsilon^k d_k,\end{aligned}$$

where ω_0, d_0 are solutions when $\alpha = \epsilon = \beta = 0$.

Using the theory of perturbations, the equation of motion is, up to first order in ϵ,

$$\left.\begin{aligned}&\omega_0\,\Delta d_0 + \frac{1}{r}\,\partial_r\phi\,.\,\partial_\theta\Delta d_0 + \frac{K^2}{r}\,\partial_r\phi\partial_\theta d_0 = 0,\\ &\omega_0\,\Delta d_1 + \omega_1\,\Delta d_0 + \frac{1}{r}\,\partial_r\phi\,\partial_\theta\Delta d_1 + \frac{K^2}{r}\,\partial_r\phi\,\partial_\theta d_1 - \alpha'(\Delta + K^2)\,\partial_x d_0 = 0.\end{aligned}\right\} \qquad (7)$$

In the first order calculation, the eddy is the only important quantity: we use the first equation in (7). This is equivalent to taking $\epsilon = 0, f = f_0$. The second order treatment ($\epsilon \neq 0, \alpha \neq 0$) is in §4.

The 'memory' of the current is now in the explicit shape of the eddy only. We call

$$\omega_0 = \Omega_r + i\Omega_i, \quad d_0 = \exp(im\theta)\,\xi(r) \tag{8}$$

(this is not a limitation because the equations are linear); since $\partial_r \phi \neq 0$, from (7) we have

$$\left\{1 + \frac{r}{m\,\partial_r\phi}\,\Omega_i - i\,\frac{r}{m\,\partial_r\phi}\,\Omega_r\right\}\Delta d_0 = -K^2 d_0. \tag{9}$$

We impose regularity at the origin and $d_0 = 0$ at $\partial\mathscr{C}$, $r = R_0$; i.e. we study perturbations with scales smaller than the eddy.

We now show directly that this restricted class of perturbations is stable: $\Omega_r = 0$.

Let us assume that $\Omega_r \neq 0$. We can then multiply the preceding equation by the finite quantity

$$\left(1 + \frac{r\Omega_i}{m\,\partial_r\phi} - i\,\frac{r\Omega_r}{m\,\partial_r\phi}\right)^{-1} . d_0^*$$

thus obtaining

$$d_0^*\,\Delta d_0 = -K^2\left(1 + \frac{r\Omega_i}{m\,\partial_r\phi} - i\,\frac{r\Omega_r}{m\,\partial_r\phi}\right)^{-1} d_0^*\,d_0.$$

We now integrate in space:

$$\begin{aligned} -K^2\int_{r\leqslant R_0} dx\,dy\,\frac{d_0^*\,d_0}{1 + r\Omega_i/m\,\partial_r\phi - i\,r\Omega_r/m\,\partial\phi} &= \int_{r\leqslant R_0} dx\,dy\,d_0^*\,\Delta d_0 \\ &= -\int_{r\leqslant R_0} dx\,dy\,(\nabla d_2^*)\times(\nabla d_0) \leqslant 0. \end{aligned}$$

If the kinetic energy of our bounded system is finite, the left-hand integral is a negative real quantity. This implies

$$\Omega_r \int_{r\leqslant R_0} dx\,dy\,\frac{r}{\partial_r\phi}\,\frac{d_0^*\,d_0}{(1 + r\Omega_i/m\partial_r\phi)^2 + (r\Omega_r/m\,\partial_r\phi)^2} = 0.$$

This contradicts our hypothesis $\Omega_r \neq 0$. Thus $\Omega_r = 0$ and the system is stable. This is a rigorous result in a central region, i.e. $r \leqslant R_o$, but it could also be true everywhere.

The same result holds if the integral is performed over any region bounded by a $d_0 = 0$ line, as long as $\partial_r\phi \neq 0$ inside the region.

The equation is then

$$(1 + r\Omega_i/m\partial_r\phi)\,\Delta d_0 = -K^2 d_0. \tag{10}$$

This equation is discussed in the appendix.

4. The effect of the current on eddy stability

The effect of the current $\alpha(y)$ is now discussed. The order ϵ stability equation (7) gives:

$$\omega_1\Delta d_0 + \omega_0\Delta d_1 + \frac{1}{r}\,\partial_r\phi\partial_\theta\,\Delta d_1 + \frac{1}{r}\,K^2\,\partial_r\phi\,\partial_\theta d_1 - \alpha'(\Delta + K^2)\,\partial_x d_0 = 0, \tag{11}$$

where $\omega_0 = i\Omega_i$.

It will now be shown that $\omega_1 = -\omega_1^*$. A slightly modified version of the second order perturbation theory is used, multiplying equation (11) by

$$\frac{d_0^*}{\Omega_i + (1/r)\, m\, \partial_r \phi} = \frac{r}{\partial_r \phi}\, \Delta d_0^*$$

and integrating over the region $0 \leqslant r \leqslant Z_0$. Here Z_0 is the smallest zero of the

$$1 + r\Omega_i/m\,\partial_r \phi$$

function: i.e.

$$1 + Z_0\,\Omega_i/m\,\partial_r \phi(Z_0) = 0.$$

One could also see from (10) that $d(Z_0) = 0$ (see appendix). We then obtain

$$0 = \int_{r\leqslant Z_0} dx\,dy\, \frac{d_0^*}{\Omega_i + (m/r)\,\partial_r\phi} \Big\{ i\Omega_i \Delta d_1 + \omega_1 \Delta d_0 + \frac{1}{r}\,\partial_r\phi\partial_\theta \Delta d_1 + \frac{1}{r}\, K^2\, \partial_r\phi\partial_\theta d_1 - \alpha'(\Delta + K^2)\,\partial_x d_0 \Big\}.$$

Some general properties of partial integration are used, i.e. the fact that $\partial_\theta \phi(r) = 0$ and that $\Delta\,.\,\partial_\theta = \partial_\theta\,.\,\Delta$. This gives:

$$0 = \int_{r\leqslant Z_0} dx\,dy \left\{ \left(\Delta d_0^* + \frac{K^2}{1 + r\Omega_i/m\,\partial_r\phi}\, d_0^* \right) i d_1 + \omega_1 \frac{d_0^*\, \Delta d_0}{\Omega_i + (m/r)\,\partial_r\phi} - \frac{d_0^*\, \alpha'(\Delta + K^2)\,\partial_x d_0}{\Omega_i + (m/r)\,\partial_r\phi} \right\}.$$

The first two terms on the left-hand side of the integral compensate each others exactly because of the properties of (10) so we have

$$\begin{aligned}
\omega_1 \int_{r\leqslant Z_0} dx\,dy\, \frac{d_0^*\, \Delta d_0}{\Omega_i + (m/r)\,\partial_r\phi} &= \int_{r\leqslant Z_0} dx\,dy\, \frac{d_0^*\, \alpha'(\Delta + K^2)\,\partial_x d_0}{\Omega_i + (m/r)\,\partial_r\phi} \\
&= \int_{r\leqslant Z_0} dx\,dy\, \frac{d_0^*\, \alpha'\, \partial_x(\Delta + K^2)\, d_0}{\Omega_i + (m/r)\,\partial_r\phi} \\
&= \int_{r\leqslant Z_0} \frac{r\,dr\,d\theta}{\Omega_i + (m/r)\,\partial_r\phi} \Big\{ \xi^*(r) \exp(-im\theta)\, \alpha'(y) \\
&\quad \times \partial_x \left[K^2 \exp(im\theta)\, \xi(r) \left(1 - \frac{1}{1 + r\Omega_i/m\,\partial_r\phi} \right) \right] \Big\}
\end{aligned}$$

because of (10) and (7). Calling

$$F(r) = K^2 \xi(r) \left(1 - \frac{1}{1 + r\Omega_i/m\partial_r\phi} \right)$$

we have

$$\begin{aligned}
\omega_1 \int_{r\leqslant Z_0} & dx\,dy\, \frac{d_0^*\, \Delta d_0}{\Omega_i + (m/r)\,\partial_r\phi} \\
&= \int_{r\leqslant Z_0} \frac{r\,dr\,d\theta}{\Omega_i + (m/r)\,\partial_r\phi} \,.\, \xi^*(r) \exp(-im\theta)\, \alpha'(y)\, \partial_x(\exp(im\theta)\, F(r)) \\
&= \int_{r\leqslant Z_0} \frac{r\,dr\,d\theta}{\Omega_i + (m/r)\,\partial_r\phi}\, \xi^*(r)\, \alpha'(y) \left\{ \cos\theta\, \partial_r F(r) - im \frac{\sin\theta}{r}\, F(r) \right\} \\
&= \int_{r\leqslant Z_0} \frac{r\,dr\,d\theta}{\Omega_i + (m/r)\,\partial_r\phi}\, \xi^*(r)\, \alpha'(y) \left\{ \frac{x}{r}\, \partial_r F(r) - imy\, \frac{F(r)}{r^2} \right\}.
\end{aligned}$$

The first integral of F is zero, being the integral of a function of r and of y multiplied by x, over a symmetric domain around the y axis. Then the real part of ω_1 is also zero as in the $\epsilon = 0$ case.

To summarize we can say that the shape of our steady solution can be varied by small time dependent perturbations. The system is moreover stable to the restricted class of perturbations with dimensions smaller than the eddy. A further nonlinear study of stability à la Liaponov is now in progress.

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Appendix

Some properties of the solution of (9), in the case $\Omega_r = 0$ are now discussed, inside the $\mathscr{C}$ region, where $\partial_r \phi \neq 0$.

The equation is now

$$\begin{cases} \left(1+\dfrac{r\Omega_i}{m\,\partial_r\phi}\right)\left(\dfrac{1}{r}\,\partial_r r\,\partial_r \xi(r) - \dfrac{m^2}{r^2}\,\xi(r)\right) = -K^2\xi(r) \\ \partial_r \xi = 0, \quad r = 0 \\ \xi = 0, \quad r = R_0. \end{cases}$$

Clearly the quantity $1+r\Omega_i/m\,\partial_r\phi$ plays an important role in this equation. If Ω_i/m is positive or negative, large or small, the quantity $1+r\Omega_i/m\,\partial_r\phi$ has positive or negative values. Its zeros Z_n define intervals I_n, in the $0 \leqslant r < \infty$ axis, where

$$1+r\Omega_i/m\,\partial_r\phi$$

is alternatively positive or negative. It can be shown (Jorgens 1970) that this equation has a solution if and only if $\zeta(r)$ is zero at $r = Z_n$.

So let us assume that in the most central interval, I_0 $(0 \leqslant r \leqslant Z_0)$ the function is positive. In the next interval I_1 $(Z_0 \leqslant r \leqslant Z_1)$ $1+r\Omega_i/m\,\delta_r\phi$ is negative and also

$$-K^2\int_{r\in I_1} dx\,dy\,\frac{d_0^*\,d_0}{1+r\Omega_i/m\,\partial_r\phi} = -\int_{r\in I_1} dx\,dy(\nabla d_0^*)\times(\nabla d_0).$$

Then either d_0 has an infinite kinetic energy or we must assume that $d_0 = 0$ in the I_1 interval. By repeating this description for various I_n, we can be sure that $d_0 \neq 0$ only in the intervals where $1+r\Omega_i/m\,\partial_r\phi > 0$ and that $d_0 = 0$ at $r = Z_n$.

The equation is then

$$\begin{cases} \left(1+\dfrac{r\Omega_i}{m\,\partial_r\phi}\right)\left(\dfrac{1}{r}\,\partial_r r\,\partial_r \xi - \dfrac{m^2}{r^2}\,\xi\right) = -K^2\xi \\ \delta_r \xi = 0, \quad r = 0 \\ \xi = 0, \quad r = Z_0 \leqslant R_0 \end{cases}$$

for the most central region.

The explicit values of Ω_i and $\xi(r)$ can eventually be found numerically.

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